

Thornton & Marion 5th Edition Problem 2 – 47

Consider a particle moving in the region $x > 0$ under the influence of a potential

$$U(x) = U_0 \left(\frac{\alpha}{x} + \frac{x}{\alpha} \right)$$

where $U_0 = 1 \text{ J}$ and $\alpha = 2 \text{ m}$. Plot the potential, find the equilibrium points and determine whether they are maxima or minima.

Define the function:

$$Ux = Uzero * \left(\left(\frac{\alpha}{x} \right) + \left(\frac{x}{\alpha} \right) \right)$$

$$Uzero \left(\frac{x}{\alpha} + \frac{\alpha}{x} \right)$$

$$Uzero = 1$$

$$\alpha = 2$$

FindMaximum[Ux, {x, 2}]

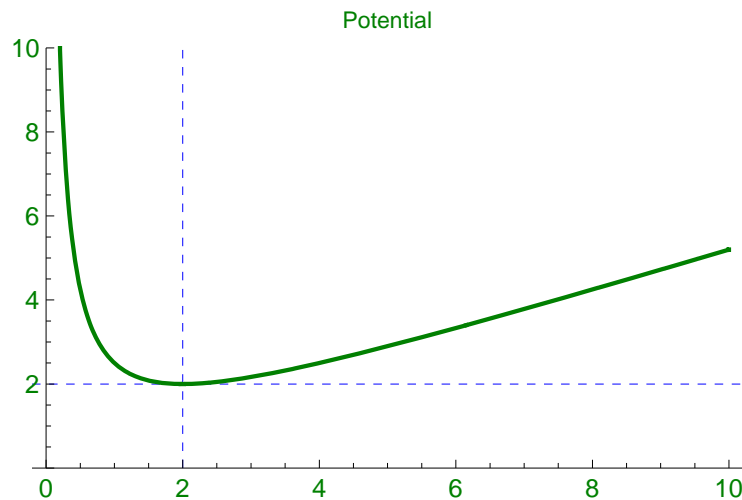
1

2

{2., {x → 2.}}

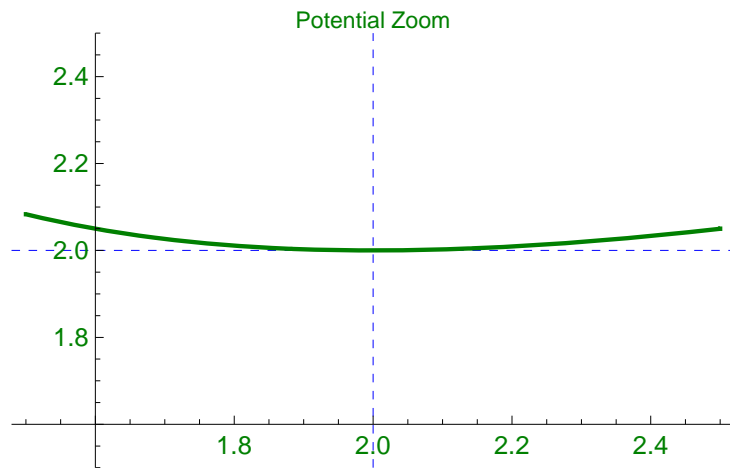
Plot the potential and include gridlines for the positions of the maxima and the values of the maxima

```
pPotential = Plot[Ux, {x, 0, 10}, TextStyle → {FontFamily → Helvetica, FontSize → 12, FontColor →
  GridLinesStyle → Directive[Blue, Dashed], PlotPoints → 500, PlotStyle → {RGBColor[0, 0.5, 0], T
```



Zoom in on the minimum to see the details.

```
pPotentialZoom = Plot[Ux, {x, 1.5, 2.5},
  TextStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[0, 0.5, 0]},
  GridLines -> {{2}, {2}}, GridLinesStyle -> Directive[Blue, Dashed],
  PlotPoints -> 500, PlotStyle -> {RGBColor[0, 0.5, 0], Thickness[0.006]},
  PlotRange -> {1.5, 2.5}, PlotLabel -> "Potential Zoom"]
```



So there is one minimum at $x = 2$ and it is a point of stable equilibrium.